Comment on 'Schwarzschild Black Hole in Noncommutative Spaces'

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abstract

A brief comment on the paper(arXiv: hep-th/0508051)with the mentioned title by F. Nasseri.

Recently there has been considerable interest in the possible effects of the noncommutative space[1, 2, 3, 4, 5, 6]. In the noncommutative quantum mechanics[7], one has assumed the commutation rules as follows

$$
\begin{aligned}
&[\hat{x}^i, \hat{x}^j] = i\theta_{ij} \ , \\
&[\hat{x}_i, \hat{p}_j] = i\delta_{ij} \ , \\
&[\hat{p}_i, \hat{p}_j] = 0 \ .
\end{aligned} \tag{1}
$$

and the coordinate transformation

$$
\begin{aligned}
&x_i = \hat{x}_i + \frac{1}{2}\theta_{ij}\hat{p}_j \ , \\
&p_i = \hat{p}_i \ .
\end{aligned} \tag{2}
$$

where the new variables satisfy the usual canonical commutation rules of quantum mechanics:

$$
\begin{aligned}
&[x_i, x_j] = 0 \ , \\
&[x_i, p_j] = i\delta_{ij} \ , \\
&[p_i, p_j] = 0 \ .
\end{aligned} \tag{3}
$$

By using these commutation rules, the author of Ref.[8] argued that the Schwarzschild black hole can be extended to the noncommutative space. Here, the basic concept were thrown into confusion because the Schwarzschild external metric is a classical solution which has nothing to do with the commutation rules of quantum mechanics in the noncommutative or commutative space. In fact, one has to develope the modified Poisson brackets of classical phase space $(\tilde{x}_i, \tilde{p}_i)$ if one would like to consider that classical metric extends to the noncommutative case. Let us now infer the Poisson brackets from the rules of commutation of noncommutatitive quantum mechanics (1),

$$
\begin{aligned}
&\{\tilde{x}_i, \tilde{x}_j\} = \theta_{ij} \ , \\
&\{\tilde{x}_i, \tilde{p}_j\} = \delta_{ij} \ , \\
&\{\tilde{p}_i, \tilde{p}_j\} = 0
\end{aligned} \tag{4}
$$

and the modified Poisson brackets for two arbitrary functions F and G defined on the phase space

$$
\{F, G\} = \theta_{ij}\frac{\partial F}{\partial \tilde{x}_i}\frac{\partial G}{\partial \tilde{x}_j} + (\frac{\partial F}{\partial \tilde{x}_i}\frac{\partial G}{\partial \tilde{p}^{\,i}} - \frac{\partial F}{\partial \tilde{p}_i}\frac{\partial G}{\partial \tilde{x}^i}) \tag{5}
$$

where the constant parameters θ_{ij} of the noncommutativity is real and antisymmetric, which has dimension of area.

In our formulation for noncommutative black hole, one can still use the usual definition for the metric

$$
ds^2 = f(\tilde{r})dt^2 - \frac{d\tilde{r}^2}{f(\tilde{r})} - \tilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{6}
$$

where $\tilde{r}$ satisfies Eq.(4). However, one should be aware that there is no modified Einstein equation in this case. In our approach, since the noncommutativity parameter, if it is non-zero, should be very small compared to the length

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scales of the black hole (for example, the horizon of black hole), one can always treat the noncommutative effects as some perturbations of the commutative counter-part and therefore one can use the usual metric. $f(\tilde{r})$ in terms of the noncommutative coordinates $\tilde{x}_i$ is:

$$f(\tilde{r}) = 1 - \frac{2GM}{\sqrt{\tilde{x}_i \tilde{x}^i}} \, . \tag{7}$$

We note that there is also a new coordinate system

$$\begin{aligned} x_i &= \tilde{x}_i + \frac{1}{2}\theta_{ij}\tilde{p}_j \, , \\ p_j &= \tilde{p}_j \, , \end{aligned} \tag{8}$$

where the new variables satisfy the usual Poisson brackets:

$$\{F, G\} = \frac{\partial F}{\partial x_i}\frac{\partial G}{\partial p^i} - \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial x^i} \tag{9}$$

and

$$\begin{aligned} \{x_i, x_j\} &= 0 \, , \\ \{x_i, p_j\} &= \delta_{ij} \, , \\ \{p_i, p_j\} &= 0 \, . \end{aligned} \tag{10}$$

Using the new coordinates, we have

$$\begin{aligned} f(r) &= 1 - \frac{2GM}{\sqrt{(x_i - \theta_{ij}p^j/2)(x_i - \theta_{ik}p^k/2)}} \\ &= 1 - \frac{2GM}{\sqrt{r^2 - \frac{(\vec{p}\times\vec{\theta})\cdot\vec{r}}{4} + \frac{\vec{p}^2\cdot\vec{\theta}^2 - (\vec{p}\cdot\vec{\theta})^2}{16}}} \\ &= 1 - \frac{2GM}{\sqrt{r^2 - \frac{\vec{L}\cdot\vec{\theta}}{4} + \frac{\vec{p}^2\cdot\vec{\theta}^2 - (\vec{p}\cdot\vec{\theta})^2}{16}}} \end{aligned} \tag{11}$$

where $\theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta_k$ and $\vec{L} = \vec{r} \times \vec{p}$. It is worth noting that the terms of above function are not only the powers in θ but also the same powers in momenta. The horizon $\tilde{r}_h$ of the noncommutative metric (6) is

$$\tilde{r}_h = [4(GM)^2 + \frac{\vec{L}\cdot\vec{\theta}}{4} + \frac{\vec{p}^{\,2}\cdot\vec{\theta}^{\,2} - (\vec{p}\cdot\vec{\theta})^2}{16}]^{\frac{1}{2}} \, , \tag{12}$$

which satisfies

$$f(\tilde{r}_h) = 0 \, . \tag{13}$$

Because Schwarzschild external metric is a solution for the gravitational field surrounding a non-rotating mass, Eq.(12) should be reduced to

$$\tilde{r}_h = [4(GM)^2 + \frac{\vec{p}^2\cdot\vec{\theta}^2 - (\vec{p}\cdot\vec{\theta})^2}{16}]^{\frac{1}{2}} \, . \tag{14}$$

The Hawking temperature and the horizon area of Schwarzschild black hole in noncommutative spaces are respectively $T_H = \frac{GM}{2\pi\tilde{r}_h^2}$ and $A = 4\pi\tilde{r}_h^2$. Obviously, they depend on the phase coordinate p_i because of the effect of noncommutative space. For a rest observer, $p_i = 0$ so that $\tilde{r}_h = 2GM$, and the horizon area increases with the momentum $|\vec{p}|$. In the limit $\theta_i \to 0$, the horizon is reduced to one in commutative space.

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